MILLIMAN REPORT

Risk-based rebalancing thresholds

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Table of Contents

1. Background

1.1 INTRODUCTION

Most investment strategies require some form of rebalancing to achieve stated objectives and remain effective as financial markets move. Milliman currently manages more than $143 billion of assets globally (as of June 30, 2020) through our investment advisory, hedging and consulting services. The bulk of the strategies managed include dynamic hedging of option-like payoff structures, for example, in the guaranteed payouts of variable annuities and replication of options within the Milliman Managed Risk Strategy.

Given the dynamic nature of risks involved, the implementation of these strategies requires risk exposures to be monitored and rebalanced frequently. The rebalancing decisions are typically governed by a set of predefined trading thresholds and logic. These thresholds and trading logic are generally designed to strike a balance between the effectiveness of the strategy meeting its objectives and transaction costs incurred from the execution of rebalancing trades.

A common practice within the industry has been to trigger rebalancing trades when the risk exposures managed breach a fixed percentage-based trading threshold or fixed dollar-based thresholds. The primary advantage of this approach lies within its simplicity and ease of execution.

In this paper, we propose a risk-based rebalancing approach where thresholds are set based on the overall portfolio's risk level. In addition, the triggered rebalancing trades are optimised based on reduction in portfolio risk versus expected transaction costs. We compared this approach against the conventional percentage-based thresholds through a hypothetical dynamic hedging program for a Japanese variable annuity (VA) portfolio.

1.2 KEY FINDINGS

We observe that the risk-based rebalancing threshold approach explored in this paper addresses the deficiencies of percentage-based thresholds, while maintaining much of the simplicity associated with this conventional approach.

Across back-tests and stochastic simulations, the risk-based rebalancing threshold approach achieved similar levels of hedge effectiveness at a cost that was approximately **30%** lower than the corresponding percentage-based threshold strategy.

2. Overview of rebalancing strategies

2.1 INTRODUCTION TO VA RISKS

VA products are a class of investment products with an embedded guarantee that offers policyholders protection for their investments. As a result, they are exposed to a variety of market risks that can be hedged using capital market instruments. VA writers commonly implement dynamic hedging programs to protect earnings and minimise the volatility of profit and losses arising from capital market movements.

For the purpose of this paper, the market risks included in the dynamic hedging program are:

- **Delta:** Sensitivity of the VA liability to equity and bond returns
- **Rho:** Sensitivity of the VA liability to changes in interest rate levels

To manage these risks, a hedge portfolio consisting of a combination of futures contracts and interest rate swaps is dynamically rebalanced to match delta and rho sensitivities above. The hedge portfolio is therefore expected to offset changes in the VA liability caused by equity, bond and interest rate movements.

In this section, we walk through two approaches in determining when and how this hedge portfolio is rebalanced.

2.2 PERCENTAGE-BASED APPROACH TO TRIGGERS AND TRADING

Under this approach, a threshold on the upper limit of percentage mismatch in risk exposure is set individually for each risk bucket. In practice, the same threshold level is commonly used across all risk exposures. When a rebalance threshold is exceeded, there is some discretion in the rebalance amount, typically such that the net risk after rebalancing is less than half of the threshold.

The rationale behind this approach is that for any individual risk exposure, the maximum tolerance is based on a fixed percentage of the VA liability's sensitivity to that risk exposure.

Whilst simple to understand and implement, some drawbacks to this approach are as follows:

1. **Overall risk level of the portfolio is not considered:** Rebalancing can be triggered by an isolated breach in a small risk factor, but not by a broader drift in multiple risk factors. This means that the overall risk of the portfolio at the time of each rebalance is not consistent and can result rebalancing trades being triggered when the overall risk is immaterial.

   In the following example, we illustrate two scenarios. In the first scenario, we observe an isolated breach in the US Treasury risk exposure, with all other risk factors closely hedged. In the second scenario, we observe all risk factors sitting close to each of their individual percentage-based thresholds but just within them.

FIGURE 1: ILLUSTRATION OF INCONSISTENT PORTFOLIO RISK LEVELS, PERCENTAGE-BASED THRESHOLD

Risk Factor	Liability Greek	Mismatch (Scen 1)		Mismatch (Scen 2)	
		in Yen	in %	in Yen	in %
Nikkei 225	-¥86,760,000.0	¥0.0	0.0%	-¥3,470,400.0	4.0%
S&P 500	-¥28,920,000.0	¥0.0	0.0%	-¥1,156,800.0	4.0%
Euro STOXX 50	-¥28,920,000.0	¥0.0	0.0%	-¥1,156,800.0	4.0%
10yr US T-Note	-¥86,760,000.0	-¥4,338,000.0	5.0%	¥3,470,400.0	-4.0%
10yr Bund	-¥86,760,000.0	¥0.0	0.0%	¥3,470,400.0	-4.0%
10yr JGB	-¥260,280,000.0	¥0.0	0.0%	¥10,411,200.0	-4.0%
JYSW15	-¥33,260,000.0	¥0.0	0.0%	-¥1,330,400.0	4.0%
JYSW20	-¥20,120,000.0	¥0.0	0.0%	-¥804,800.0	4.0%
JYSW30	-¥20,760,000.0	¥0.0	0.0%	-¥830,400.0	4.0%
Est 1-Day Volatility		**¥1,215,679.6**		**¥15,309,921.4**	
Trading Threshold		**Breached**			

Despite the first scenario having an overall portfolio risk that is more than 12 times smaller than the second scenario, it is the first scenario that will trigger a rebalancing trade.

2. **Correlation between risk factors:** Most market risk factors exhibit a high degree of correlation. Hence, being over- or under-weight in certain risk exposures can help to reduce the overall risk of the portfolio. Following a percentage-based threshold can occasionally lead to some rebalancing trades increasing the overall risk of the portfolio if conducted in isolation.

   In the example below, we illustrate a scenario where the portfolio has long exposure in 20-year rates that is currently breaching the percentage-based threshold and a short exposure in 30-year rates that is just under the percentage-based threshold.

FIGURE 2: ILLUSTRATION OF OFFSETTING IMPACT BETWEEN RISK EXPOSURES

Risk Factor	Liability Greek	Mismatch (Scen 1)		Mismatch (Scen 2)	
		in Yen	in %	in Yen	in %
Nikkei 225	-¥86,760,000.0	¥0.0	0.0%	¥0.0	0.0%
S&P 500	-¥28,920,000.0	¥0.0	0.0%	¥0.0	0.0%
Euro STOXX	-¥28,920,000.0	¥0.0	0.0%	¥0.0	0.0%
10yr US T-N	-¥86,760,000.0	¥0.0	0.0%	¥0.0	0.0%
10yr Bund	-¥86,760,000.0	¥0.0	0.0%	¥0.0	0.0%
10yr JGB	-¥260,280,000.0	¥0.0	0.0%	¥0.0	0.0%
JYSW15	-¥33,260,000.0	¥0.0	0.0%	¥0.0	0.0%
JYSW20	-¥20,120,000.0	¥1,006,000.0	5.0%	¥0.0	0.0%
JYSW30	-¥20,760,000.0	-¥830,400.0	4.0%	-¥830,400.0	4.0%
Est 1-Day Volatility		**¥662,199.0**		**¥2,480,148.9**	
Trading Threshold		**Breached**			

In this scenario, the percentage-based approach will trigger a trade in just the 20-year bucket. However, as a result of the expected correlation benefit between the 20-year and 30-year rates, by trading this, the overall risk of the portfolio is actually expected to increase.

2.3 PROPOSED APPROACH

To improve upon the percentage-based approach, we have proposed a risk-based rebalancing threshold that triggers rebalancing trades only when the overall risk of the portfolio has hit a certain risk level.

This approach ensures that each time a rebalancing trade is triggered, it is triggered when the overall portfolio is at a material and consistent risk level. Further, once the trade is triggered, we have proposed an optimised approach in determining which instruments to trade, taking into account the expected transaction costs and expected reduction in risk.

Risk-based triggers

A portfolio measure of risk can be constructed by aggregating individual risk exposures based on their volatility and correlations with other risk factors. This approach takes into account interaction between risks and any inherent differences in risk levels to ensure that the overall risk exceeds a similar level each time the portfolio is rebalanced. Further details on this calculation are given in the appendix.

Figure 3 shows the portfolio risk level generated with a mismatch equal to the percentage-based thresholds defined at 5%. This highlights that despite having the same 5% threshold across risk factors, a breach in the percentage-based threshold for Nikkei has more than 10 times the risk generated from a breach in the 2-, 5, 7-year interest rate buckets.

Note that the portfolio risk calculated here are expected to change depending on the volatility environment of each of these risk factors. The numbers here are representative of the market environment on 30 August 2018:

FIGURE 3: EQUIVALENT PORTFOLIO RISK WHEN THE MISMATCH IS EQUAL TO THE 5% PERCENTAGE-BASED THRESHOLDS

RISK TYPE	RISK FACTOR	RISK WEIGHT	PERCENTAGE-BASED REBALANCING THRESHOLD: % THRESHOLD	PERCENTAGE-BASED REBALANCING THRESHOLD: EQUIVALENT PORTFOLIO RISK (¥)
Delta	Nikkei	15%	5%	6.7M
	S&P 500	5%	5%	1.6M
	EuroStoxx 50	5%	5%	2.0M
	10 Yr US Treasury	15%	5%	1.2M
	10 Yr Euro Bund	15%	5%	0.9M
	10 Yr JGB	45%	5%	2.4M
Rho (JPY)	Maturities 2, 5, 7 years	11%	5%	0.4M
	Maturities 10, 15, 20, 30 years	89%	5%	2.5M

With this in mind, the proposed approach is to replace the percentage-based thresholds defined for each risk factor with a single threshold set at the portfolio risk level. The analysis in Section 3 illustrates that the hedge effectiveness achieved using a 5% percentage-based threshold can be matched with portfolio level risk threshold set at ¥8 million.

In practice, absolute limits can still be put in place for individual risks to ensure that the portfolio is not overly exposed to a rapid uplift in volatility of the risk factor. This is expected to marginally reduce the benefits of this approach, but provide risk teams with more comfort in the absolute exposures allowed in each risk factor. A brief comparison of results under this approach is provided in Section 4.1.2.

Optimisation of rebalancing trades under risk-based triggers

Typical expected transaction costs (explicit and expected implicit) between market instruments can vary significantly as well.

In general, futures contracts are observed to incur lower transaction costs compared to interest rate swaps. This suggests that selectively rebalancing hedge assets that are less costly to trade can reduce overall transaction costs from the dynamic hedging program. To take advantage of cost variations between hedge assets, we allow a degree of flexibility in the rebalancing process by specifying a target level of risk after rebalancing. In the example here, we have set the target level of risk after rebalancing to be 75% of the risk-based threshold.

We propose an optimisation procedure where each rebalancing trade is ranked based on its estimated risk reduction to the overall portfolio risk and the transaction costs expected. Trades with the highest contribution to risk reduction versus expected transaction costs are rebalanced first, and this iterative process is repeated until the overall portfolio risk falls below the target level.

Note that the risk reduction calculation illustrated here assume breaches in each risk factor in isolation. In practice, the estimated risk reduction would be different due to correlation between mismatches in each of the asset classes. In fact, estimated risk reduction could be negative when certain risk factors are acting as an offset for others.

FIGURE 4: TYPICAL TRANSACTION COSTS AND ESTIMATED RISK REDUCTION BASED ON HISTORICAL VOLATILITY

		IMPACT OF ¥1,000 NOMINAL REBALANCE	
RISK TYPE	RISK FACTOR	ESTIMATED RISK REDUCTION (¥)	ESTIMATED TRANSACTION COSTS (¥)
Delta	Nikkei	1,500	4.0
	S&P 500	1,100	5.6
	EuroStoxx 50	1,400	17.6
	10 Yr US Treasury	280	8.1
	10 Yr Euro Bund	210	6.1
	10 Yr JGB	180	3.5
Rho (JPY)	Maturities 2, 5, 7 years	2,100	375
	Maturities 10, 15, 20, 30 years	2,800	500

3. Comparison of the rebalancing strategies

To assess the relative merits of the rebalancing strategies, we compare the approaches on the effectiveness of the hedge program and the transaction costs incurred to achieve it.

The effectiveness of dynamic hedging programs can be measured using a number of different methodologies. A research paper published by Milliman[1] outlines two common approaches, and the measure used in this paper is based on the percentage reduction in the volatility of weekly profit and losses.[2]

The transaction cost is then estimated based on the expected implicit and explicit costs of executing the rebalancing trades in normal market conditions.

The objective is to achieve a certain level of hedge effectiveness whilst minimising the transaction costs incurred to get there.

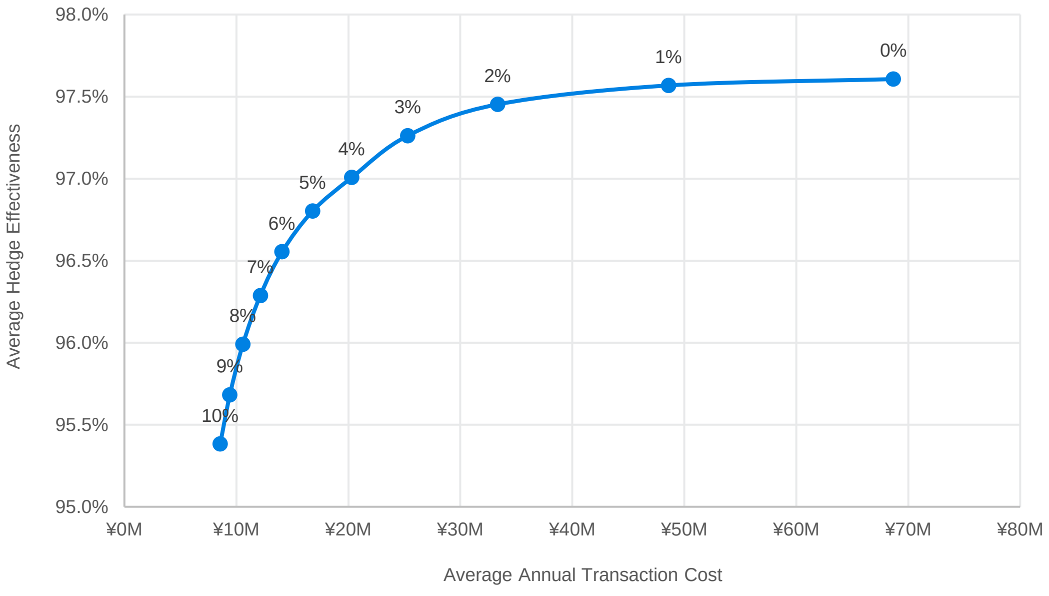
In this section, we present out-of-sample results for the two approaches using back-test and stochastic bootstrapped scenarios.

3.1 COMPARISON OF RESULTS FROM STOCHASTIC BOOTSTRAPPED SCENARIOS

Percentage-based thresholds

Figure 5 illustrates the results of a hypothetical implementation of the percentage-based threshold approach over 1,000 daily stochastic bootstrapped scenarios where each scenario spans a period of one year. We observe the trade-off between hedge effectiveness and transaction cost, where tighter thresholds result in higher hedge effectiveness at the expense of larger transaction costs from more frequent rebalancing of hedge assets. Previous Milliman research found that setting the threshold at 5% represented a good balance between competing objectives of maximising hedge effectiveness and minimising transaction costs for variable annuity hedging programs.

FIGURE 5: AVERAGE HEDGE EFFECTIVENESS AND TRANSACTION COSTS FOR PERCENTAGE-BASED THRESHOLDS



[1] See Sun, P., Kelkar, R., Dai, J., and Huang, V. (December 2016). How effective is variable annuity guarantee hedging? Milliman research report. Available at: https://au.milliman.com/-/media/milliman/importedfiles/uploadedfiles/insight/2016/variable-annuity-guarantee-hedging.ashx

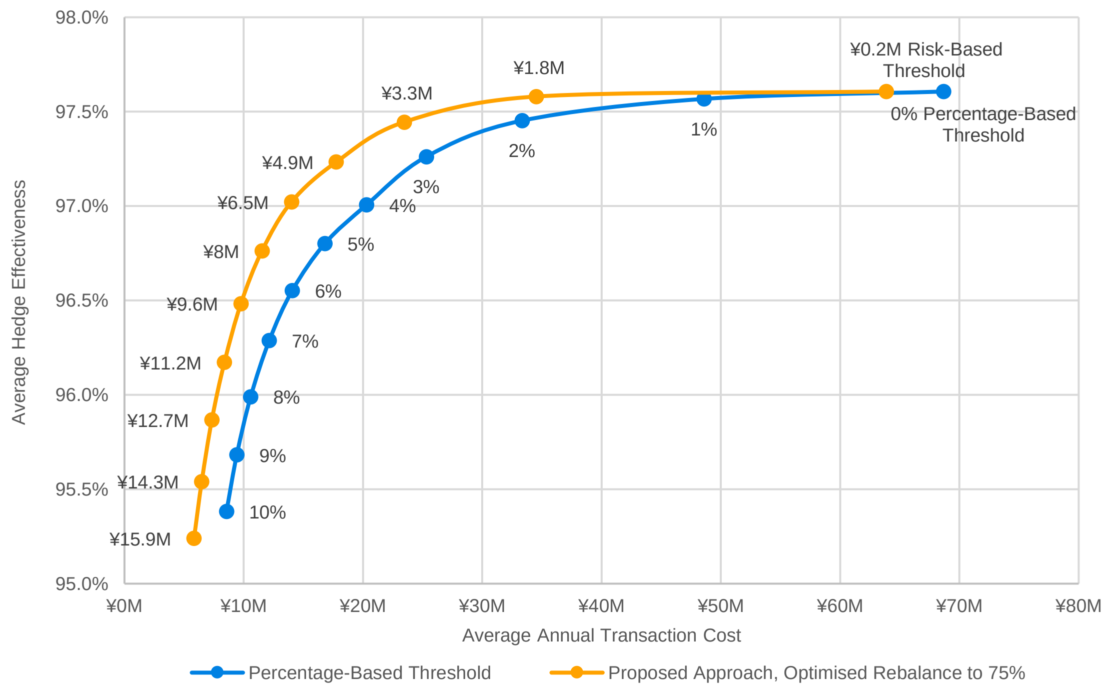
[2] Defined as $1 - \frac{\textit{Standard deviation of weekly hedged P\&L}}{\textit{Standard deviation of weekly unhedged P\&L}}$

Proposed approach

Using the same 1,000 scenarios, we compared the results from the percentage-based threshold approach against the proposed approach, where hedge assets are rebalanced until the overall portfolio risk is less than 75% of the specified threshold when rebalance thresholds are exceeded.

Across all thresholds, we found that the proposed strategy was able to achieve a similar level of hedge effectiveness while incurring lower transaction costs. In particular, the 5% percentage-based threshold achieved an effectiveness of 96.8% using ¥17 million in expected transaction costs, whilst a ¥8 million risk-based threshold achieved the same effectiveness using just ¥12 million in transaction costs. This represents a 31% reduction in transaction costs whilst maintaining the same effectiveness.

FIGURE 6: AVERAGE HEDGE EFFECTIVENESS AND TRANSACTION COSTS, STOCHASTIC BOOTSTRAPPED SCENARIOS



The efficiency generated through the proposed approach is expected. The following are the two key contributors to this result:

1. The approach is expected to eliminate a proportion of unnecessary rebalancing trades. Figure 7 illustrates the proportion of trades (by notional) triggered by the percentage-based approach when the portfolio risk level was under ¥8 million:

FIGURE 7: NOTIONAL TRADES AND ESTIMATED TRANSACTION COSTS UNDER PERCENTAGE-BASED APPROACH WHERE PORTFOLIO RISK LEVEL IS BELOW ¥8 MILLION

NOTIONAL TRADED (¥)	PROPORTION OF TOTAL	ESTIMATED ANNUAL TRANSACTION COSTS (¥)	PROPORTION OF TOTAL
135.5M	34.3%	2.6M	15.5%

2. The risk-based approach with the optimised instrument selection is expected to tilt towards trading instruments with a lower transaction cost.[3] Figure 8 illustrates the average notional of trades for each risk factor under the 5% percentage-based threshold and the ¥8 million risk-based threshold:

FIGURE 8: BREAKDOWN OF REBALANCING TRADES BY RISK FACTORS

RISK TYPE	RISK FACTOR	ESTIMATED TRANSACTION COST PER UNIT OF RISK REDUCTION (BPS)	NOTIONAL TRADED (PERCENTAGE-BASED THRESHOLD, ¥)	NOTIONAL TRADED (PROPOSED APPROACH, ¥)	CHANGE (%)
Delta	Nikkei 225	0.40	153.0M	163.9M	7%
	S&P 500	0.56	38.6M	33.4M	-14%
	EuroStoxx 50	1.76	44.1M	28.3M	-36%
	10 Yr US Treasury	0.81	25.5M	17.1M	-33%
	10 Yr Euro Bund	0.61	29.1M	14.5M	-50%
	10 Yr JGB	0.35	74.8M	40.8M	-45%
Rho	Maturities 2, 5, 7 years	37.50	3.5M	2.7M	-24%
	Maturities 10, 15, 20, 30 years	50.00	26.5M	17.6M	-33%

Comparison of results from back-test scenarios

A similar improvement was observed over back-test scenarios, as summarised in Figure 9. Our proposed approach demonstrated similar levels of hedge effectiveness and lower transaction costs across all years.

In particular, during 2020, we see a 51% reduction in transaction costs and a 97.1% effectiveness compared to 94.5% effectiveness of the percentage-based approach.

FIGURE 9: AVERAGE HEDGE EFFECTIVENESS AND TRANSACTION COSTS, BACK-TEST SCENARIOS

STRATEGY	HEDGE EFFECTIVENESS		ESTIMATED ANNUAL TRANSACTION COSTS	
	5% THRESHOLD	PROPOSED APPROACH	5% THRESHOLD	PROPOSED APPROACH
2007	96.8%	96.8%	¥14.9M	¥8.9M
2008	96.4%	95.7%	¥31.6M	¥29.1M
2009	97.4%	97.3%	¥26.3M	¥14.5M
2010	97.2%	97.2%	¥21.1M	¥15.0M
2011	97.0%	97.5%	¥14.2M	¥9.5M
2012	97.9%	97.7%	¥14.2M	¥8.1M
2013	97.5%	97.2%	¥18.5M	¥8.4M
2014	97.6%	97.2%	¥6.6M	¥5.0M
2015	96.3%	96.1%	¥14.0M	¥3.7M
2016	97.6%	97.1%	¥18.6M	¥14.9M
2017	97.6%	95.7%	¥3.8M	¥0.6M
2018	97.4%	97.3%	¥6.5M	¥2.7M
2019	97.2%	97.1%	¥7.7M	¥3.0M
2020	94.5%	97.1%	¥12.7M	¥6.2M
‘07-‘20	97.0%	96.9%	¥15.1M	¥9.3M

[3] Further details on the optimisation process are provided in Section 6.1.3 in the appendix.

4. Implementation considerations

4.1 SETTING RISK-BASED THRESHOLD LEVELS

Calibration to existing thresholds

For dynamic hedging programs currently using percentage-based thresholds, a stochastic analysis can be carried out across multiple risk-based thresholds to determine the threshold that most closely matches the existing strategy in terms of either hedge effectiveness or expected transaction costs.

The risk-based threshold approach allows for flexibility in designing hedge targets as well. For example, thresholds can be set based on a daily value-at-risk (VAR) consistent with the risk tolerance of the hedging program.

Additional thresholds on individual risk exposures

A concern with the risk-based threshold approach is that less cost-effective risk factors may be too infrequently rebalanced, potentially resulting in excessive mismatches between liability and hedge asset sensitivities. Additionally, if there is a rapid uplift in volatility of the risk factors, the portfolio risk used for the threshold could be underestimating the actual risk.

To counteract this, additional percentage-based or dollar-based thresholds can be specified on the mismatch in each risk exposure. These thresholds should be set sufficiently wide in order to ensure that they are not exceeded frequently, as this will reduce the efficiency of the risk-based threshold approach. In addition, thresholds can take into consideration the liquidity of each instrument. For example, illiquid instruments may need slightly lower thresholds so that rebalancing trades placed are not too large, incurring additional transaction costs in the process.

A comparison of hedge effectiveness and estimated transaction costs incurred under a ¥8 million risk-based threshold with 10% thresholds on each risk exposure against the proposed approach and corresponding 5% percentage-based threshold is summarised in Figure 10. We observe that hedge effectiveness is largely similar. The addition of thresholds resulted in higher transaction costs, albeit still less than that incurred under the 5% percentage-based threshold.

FIGURE 10: PROPOSED APPROACH WITH 10% THRESHOLD ON INDIVIDUAL RISK EXPOSURES AT A ¥8 MILLION RISK-BASED THRESHOLD ACROSS BOOTSTRAPPED SCENARIOS

HEDGE EFFECTIVENESS			ESTIMATED ANNUAL TRANSACTION COSTS		
5% THRESHOLD	PROPOSED APPROACH	PROPOSED APPROACH WITH 10% THRESHOLD	5% THRESHOLD	PROPOSED APPROACH	PROPOSED APPROACH WITH 10% THRESHOLD
96.80%	96.76%	96.84%	¥16.8M	¥11.5M	¥13.5M

4.2 TRADE EXECUTION STRATEGY

The rebalance target determines the rebalance amounts and frequency, where higher targets give rise to less frequent rebalancing but larger trade sizes per rebalance, and vice versa. In our example, we found no material differences in simulated results between various rebalance targets. However, for sizeable risk exposures where market impact is a concern, lower rebalance targets should be considered to allow for more frequent rebalancing with smaller trade sizes.

4.3 VOLATILITY SOURCES

In this paper, we found that the use of historical volatility was an effective measure of portfolio risk. Other potential sources could be market implied volatilities or forward-looking estimates from volatility models. We note that market-implied volatilities are not readily available and may be biased for certain illiquid instruments such as long-duration interest rates swaps and long-dated index options.

4.4 TRANSACTION COSTS

Commissions and average bid-ask spreads were included in the estimated transaction costs. Typically, trade sizes for dynamic hedging programs are not large enough to result in additional slippage from market impact. However, for applications where large trade sizes are expected, models such as those derived from the 'square root law'[4] can be used to estimate market impact costs.

5. Conclusions

Rebalancing strategies have a variety of applications in funds and risk management programs. A well-executed strategy is critical to ensure that actual and target performance are closely aligned. At the same time, transaction costs incurred can result in a material drag on performance, and consideration should be given to reducing the cost of rebalancing trades.

For portfolios that are exposed to complex and diverse risks, it is vital to ensure that they are monitored in a holistic manner. However, most rebalancing strategies focus on individual risks instead, leading to inefficiencies in the rebalancing process.

This paper demonstrates that compared to conventional percentage-based thresholds, a rebalancing strategy based on risk-based trading thresholds delivers a similar level of effectiveness at significantly lower costs. The outperformance was achieved consistently across a variety of market scenarios.

4 The square root law refers to a model described in Grinold and Kahn (1994). This commonly takes the rough form $\alpha\sigma\sqrt{\frac{V_{trade}}{V_{daily}}}$, where σ is daily volatility, V_{trade} is the intended trade size, V_{volume} is the average daily volume, and α is an instrument-specific constant calibrated using available data.

6. Appendix

6.1 MODELLING APPROACH

Scenario sets

Rebalancing strategies were projected under the following scenario sets:

1. Single back-tested scenario between 2007 and 2020/07/31.

2. 1,000 daily stochastic bootstrapped scenarios based on sampling historical daily returns and key rate movements between 2007 and 2020/07/31 in intervals of 20 trading days across all risk factors. A period of 20 trading days was chosen, as it represents the average number of trading days in a month. This is likely to be sufficiently long to capture significant periods of volatility and autocorrelation while ensuring sufficient randomness and minimising discontinuities within each scenario. Sampling was performed with replacement, and each scenario spans a period of one year. It was assumed that one year consists of 260 trading days.

Data

Historical data spanning 2002 to 2020/07/31 was collected for the following indices:

FIGURE 11: RISK FACTORS AND INDICES USED

RISK TYPE	RISK FACTOR	BLOOMBERG TICKER
Delta	Nikkei 225	NKYTR Index
	S&P 500	SPTR Index
	EuroStoxx 50	SX5T Index
	10 Yr US Treasury	LUATTRUU Index
	10 Yr Euro Bund	LBEATREU Index
	10 Yr JGB	SPJGBTR Index
Rho	Maturities 2, 5, 7 years	JYSW2 Curncy, JYSW5 Curncy, JYSW7 Curncy
	Maturities 10, 15, 20, 30 years	JYSW10 Curncy, JYSW15 Curncy, JYSW20 Curncy, JYSW30 Curncy

The Total Return indices above are used as a proxy for futures contracts commonly used in dynamic hedging programs. There may be slight differences in historical returns observed; however, this is expected to be immaterial and will not affect conclusions drawn from this analysis.

Out-of-sample results presented in this report were based on data from 2007 onwards, while data prior to 2007 formed part of the observation period used to calculate volatility and correlation assumptions for the proposed approach.

For simplicity, all risk exposures are assumed to be currency hedged.

Distributions

Annualised volatility and correlation of weekly returns and key rate movements for back-test and bootstrapped scenarios are summarised in Figures 12 and 13, respectively.

FIGURE 12: CORRELATION AND ANNUALISED VOLATILITY, BACK-TEST SCENARIOS

	Vol	Correlation												
		Delta						Rho						
Risk Factor	Annual	Nikkei	S&P	Euro Stoxx	10 Yr USD	10 Yr Bund	10 Yr JGB	2 Yr	5 Yr	7 Yr	10 Yr	15 Yr	20 Yr	30 Yr
Nikkei	25%	1	0.66	0.64	-0.35	-0.18	-0.37	0.19	0.32	0.37	0.41	0.43	0.45	0.44
S&P	18%		1	0.78	-0.38	-0.13	-0.17	0.07	0.14	0.17	0.20	0.22	0.24	0.25
EuroStoxx	23%			1	-0.42	-0.14	-0.23	0.12	0.18	0.21	0.23	0.24	0.25	0.24
10 Yr USD	5%				1	0.58	0.49	-0.30	-0.40	-0.44	-0.45	-0.44	-0.41	-0.39
10 Yr Bund	3%					1	0.43	-0.33	-0.37	-0.40	-0.40	-0.39	-0.37	-0.35
10 Yr JGB	3%						1	-0.67	-0.89	-0.93	-0.90	-0.84	-0.77	-0.70
2 Yr	20%							1	0.84	0.74	0.64	0.54	0.47	0.40
5 Yr	34%								1	0.97	0.91	0.82	0.74	0.66
7 Yr	39%									1	0.97	0.90	0.83	0.76
10 Yr	41%										1	0.97	0.92	0.86
15 Yr	43%											1	0.98	0.94
20 Yr	45%												1	0.98
30 Yr	48%													1

FIGURE 13: CORRELATION AND ANNUALISED VOLATILITY, BOOTSTRAPPED SCENARIOS

	Vol	Correlation												
		Delta						Rho						
Risk Factor	Annual	Nikkei	S&P	Euro Stoxx	10 Yr USD	10 Yr Bund	10 Yr JGB	2 Yr	5 Yr	7 Yr	10 Yr	15 Yr	20 Yr	30 Yr
Nikkei	24%	1	0.71	0.70	-0.40	-0.13	-0.33	0.18	0.31	0.35	0.39	0.42	0.44	0.45
S&P	19%		1	0.80	-0.39	-0.07	-0.19	0.09	0.17	0.21	0.25	0.29	0.31	0.32
EuroStoxx	22%			1	-0.40	-0.02	-0.18	0.07	0.16	0.20	0.23	0.28	0.29	0.31
10 Yr USD	4%				1	0.56	0.57	-0.35	-0.49	-0.54	-0.55	-0.55	-0.51	-0.49
10 Yr Bund	4%					1	0.47	-0.35	-0.43	-0.45	-0.45	-0.43	-0.40	-0.37
10 Yr JGB	3%						1	-0.64	-0.87	-0.91	-0.88	-0.80	-0.73	-0.65
2 Yr	19%							1	0.85	0.74	0.62	0.52	0.44	0.36
5 Yr	28%								1	0.97	0.89	0.79	0.70	0.61
7 Yr	32%									1	0.97	0.89	0.81	0.72
10 Yr	34%										1	0.96	0.91	0.84
15 Yr	37%											1	0.98	0.93
20 Yr	39%												1	0.97
30 Yr	44%													1

Estimation of portfolio risk level

Portfolio risk in our proposed approach is estimated as the one-day look-ahead volatility, calculated as:

$$\frac{1}{\sqrt{5}} \times \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{i,j}}$$

where w refers to the dollar mismatch between hedge asset and liability sensitivities, and $\sigma,\ \rho$ refers to the standard deviation and correlation of weekly returns and key rate movements respectively.

To ensure out-of-sample results, standard deviation and correlations used in the simulated results were calculated over non-overlapping measurement periods:

1. Back-test scenario: Rolling 5-year window preceding each simulation date
2. Bootstrapped scenarios: Period between 2002 and 2006

Optimisation process

An iterative approach is followed to determine the most cost-effective exposures to rebalance:

1. The reduction in portfolio risk level as a result of rebalancing each individual risk exposure is determined.
2. At the same time, expected transaction cost incurred to rebalance each risk exposure is calculated.
3. For each risk exposure, we derive an effectiveness measure by dividing the portfolio risk reduction in (1) by transaction costs incurred in (2).
4. The risk exposure with the combination of largest portfolio risk reduction and lowest transaction costs will have the highest effectiveness metric. This risk exposure is rebalanced and portfolio risk is recalculated.
5. This process is repeated until the portfolio risk falls below the target level.

Transaction costs

Commissions and average bid-ask spreads were included in our estimate of transaction costs and are summarised in Figure 14.

FIGURE 14: ESTIMATED TRANSACTION COSTS FOR RISK FACTORS

RISK TYPE	RISK FACTOR	ESTIMATED TRANSACTION COST PER UNIT OF RISK REDUCTION (BPS)
Delta	Nikkei 225	0.40
	S&P 500	0.56
	EuroStoxx 50	1.76
	10 Yr US Treasury	0.81
	10 Yr Euro Bund	0.61
	10 Yr JGB	0.35
Rho	Maturities 2, 5, 7 years	37.50
	Maturities 10, 15, 20, 30 years	50.00

Projection of liability value and sensitivities

Valuation of VA liabilities is typically performed by projecting the initial account value and guarantee base over risk-neutral scenarios and discounting projected claims using risk-free rates and actuarial assumptions such as mortality and lapse rates.

For simplicity, a Taylor-series approximation was used to project liability value and sensitivities along various paths. A grid of liability values was generated across a number of account value (delta) and parallel rate (rho) shocks. The projected liability value was then determined by interpolating on cumulative returns and key rate movements from the starting position.

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